DEPARTMENT OF COMMERCE AND LABOR

BULLETIN
OF THE
BUREAU OF STANDARDS

VOLUME 4
1907-8

DEPARTMENT OF COMMERCE AND LABOR
UNITED STATES OF AMERICA

DEPARTMENT OF COMMERCE AND LABOR

BULLETIN

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

VOLUME 4

(Nos. 1, 2, 3, 4)

1907-8

WASHINGTON
GOVERNMENT PRINTING OFFICE
1908

AN APPARATUS FOR DETERMINING THE FORM OF A WAVE OF MAGNETIC FLUX.

By M. G. Lloyd and J. V. S. Fisher.

The apparatus here described was designed and constructed in the course of measurements upon the magnetic losses of energy in sheet iron. Its object was the determination of the average value of an alternating electromotive force from which the form factor of the emf. wave and the maximum value of the magnetic flux inducing it can be computed. It serves equally well, however, for the tracing of a wave of magnetic induction, and that use of the apparatus is described in the present article with illustrative examples.

DESCRIPTION OF APPARATUS.

The essential part of the apparatus is a rotating commutator which reverses the contacts twice per cycle. An ebonite disk is mounted on a shaft between bearings; to its circumference is fastened a thin conducting strip with gaps at two points 180° apart. Four brushes, equally spaced, bear upon the metal rim of the rotating disk, and their joint action is that of commutation. A carrier, of brass, having four arms, is mounted loose upon the shaft; the arms support the brush holders. Screwed fast to the carrier is a gear wheel which meshes with a worm. A graduated circular scale is attached to the carrier, and is read by means of a fixed index fastened to the base.

By means of the worm and gear the brushes may be set in any desired position. The brushes are thoroughly insulated from each other and from the carrier by means of ebonite strips at the extremities of the arms.

The shaft of the rotating commutator may be coupled directly to the generator shaft. The instrument was designed for use with a four-pole machine, running at 1800 revolutions per minute.

Fig. 1 shows a photograph of the commutator in position for use. The principal dimensions are:

Circumference of disk	56	cm
Thickness of metal rim	1.5	mm
Width of gaps in rim	2	mm
Gear diameter, 96 teeth	10.2	cm

The wheel gearing with the worm on the end of the shaft makes an electrical contact for every hundred revolutions of the shaft. This serves to record the speed of rotation.

Referring to Fig. 1, the brushes *a c* are connected to a secondary coil wound around the flux to be measured. The brushes *b d* are connected to an indicating direct-current instrument, such as a d'Arsonval galvanometer or Weston voltmeter. Any instrument whose deflection is proportional to the first power of the current will answer the purpose.

To plot a curve of magnetic induction, a reading on the Weston instrument is taken for a definite position of the brushes. Then successive readings are taken, the brushes being advanced the same number of degrees each time until they have been shifted 180°. In practice it suffices to shift the brushes only 90°, corresponding to a half cycle, since the readings repeat themselves in the second quadrant when only the odd harmonics of the fundamental frequency are present, and the apparatus is suited only to waves of this character. This condition means that the positive and negative lobes of the wave shall be similar, a condition which will be fulfilled if the magnetization is produced by a well-designed and well-constructed generator.

THEORY OF THE METHOD.

Let Φ = magnetic flux to be determined.

T = its period.

e = instantaneous emf. induced in secondary coil.

N = turns in secondary coil.

t = time of commutation.

V = reading of voltmeter, to which commutated emf. is applied.

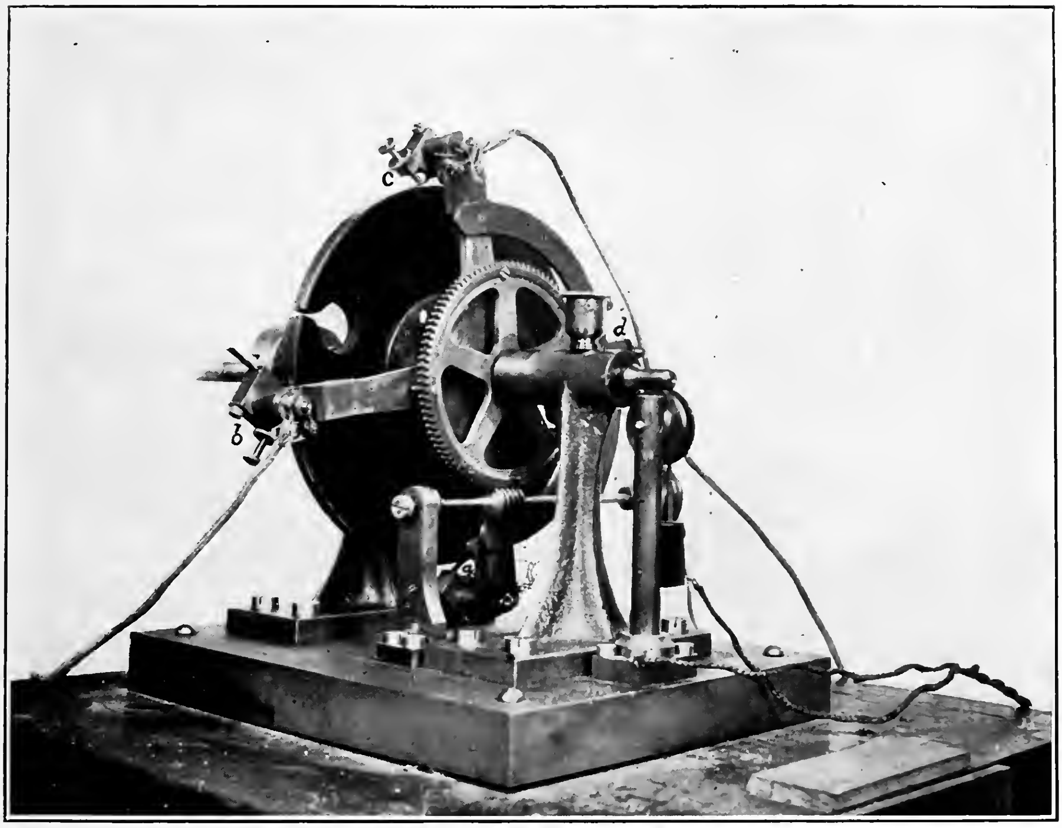

Fig. 1.—*Rotating Commutator with Adjustable Brushes for Tracing Magnetic Waves.*

Then

$$e = -N\frac{d\Phi}{dt}.$$

$\Phi = -\frac{1}{N}\int e\,dt$ is the change in flux during the time expressed by the limits of integration. In the present case the integration is carried on between two reversals or a half period and is repeated during the following half period in the reverse direction.

Hence

$$\Phi_{t+\frac{T}{2}} - \Phi_t = -\frac{1}{N}\int_t^{t+\frac{T}{2}} e\,dt$$

Now, if positive and negative lobes of the wave are similar,

$\Phi_{t+\frac{T}{2}} = -\Phi_t$ Also, $\frac{2}{T}\int_t^{t+\frac{T}{2}} e\,dt$ is the reading of the voltmeter V. Or $-2\,\Phi_t = -\frac{VT}{N2}$ and $\Phi_t = \frac{VT}{4N}$

This shows that the value of the magnetic flux at any instant is proportional to the algebraic average of the induced emf. during the succeeding half period. Since the emf. must change sign when the flux passes through a maximum, this maximum will be given by the average value of the emf. during a half period beginning with its zero value. If the wave of magnetic flux has only one maximum during a period, as is usually the case, then the emf. will remain positive during the half period next succeeding the maximum, and the average value of the emf. will be a numerical average. In this case the reading of the voltmeter will give what is commonly known as the "average value of the emf." If to the secondary coil another instrument be directly connected whose indications are proportional to the square of the emf., as a dynamometer or electrometer, this instrument will give the "effective value" of the emf. and the ratio of the two readings will give the form factor.

If, however, the flux wave be so much distorted as to include a dimple between two maxima, the emf. will change sign *during* the half period, the algebraic average will no longer be a numerical

average, and the form factor as determined above will not agree with the customary definition of form factor.

The apparatus can also be used to plot curves of electromotive force or current. For this purpose the emf., or current, is applied to the primary of an air transformer, the secondary of the same being connected through the rotating commutator to the Weston voltmeter, and readings taken as before.

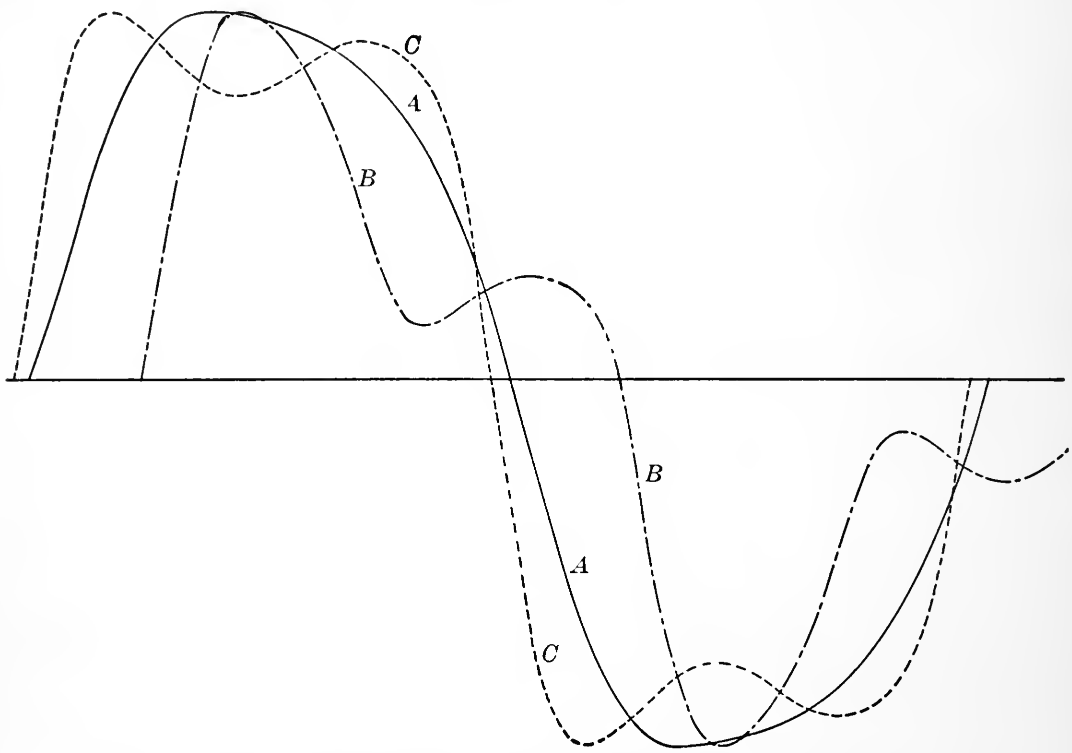

Fig. 2.—*Waves of Magnetic Flux. June 5, 1907.* B_{max}=*10,000 gausses. Period = 0.0333 sec.*

The principal source of error in the apparatus is in the imperfect contact of the brushes, and the breaking of the contact upon reversing. The contact resistance is a variable and uncertain quantity. It is desirable, therefore, to have a circuit of high resistance, so that the maximum value of the contact resistance is a negligible part of the whole. This was accomplished in the experiments cited by using a very sensitive Weston instrument, giving full scale deflection with about 0.0004 ampere. Resistance sufficiently high to make the inductance error negligible, could be connected in series.

This difficulty can be avoided by using an electrometer as indicating instrument. In this case the two pairs of quadrants are connected to the commutator brushes, while the needle is kept charged to a constant high potential.

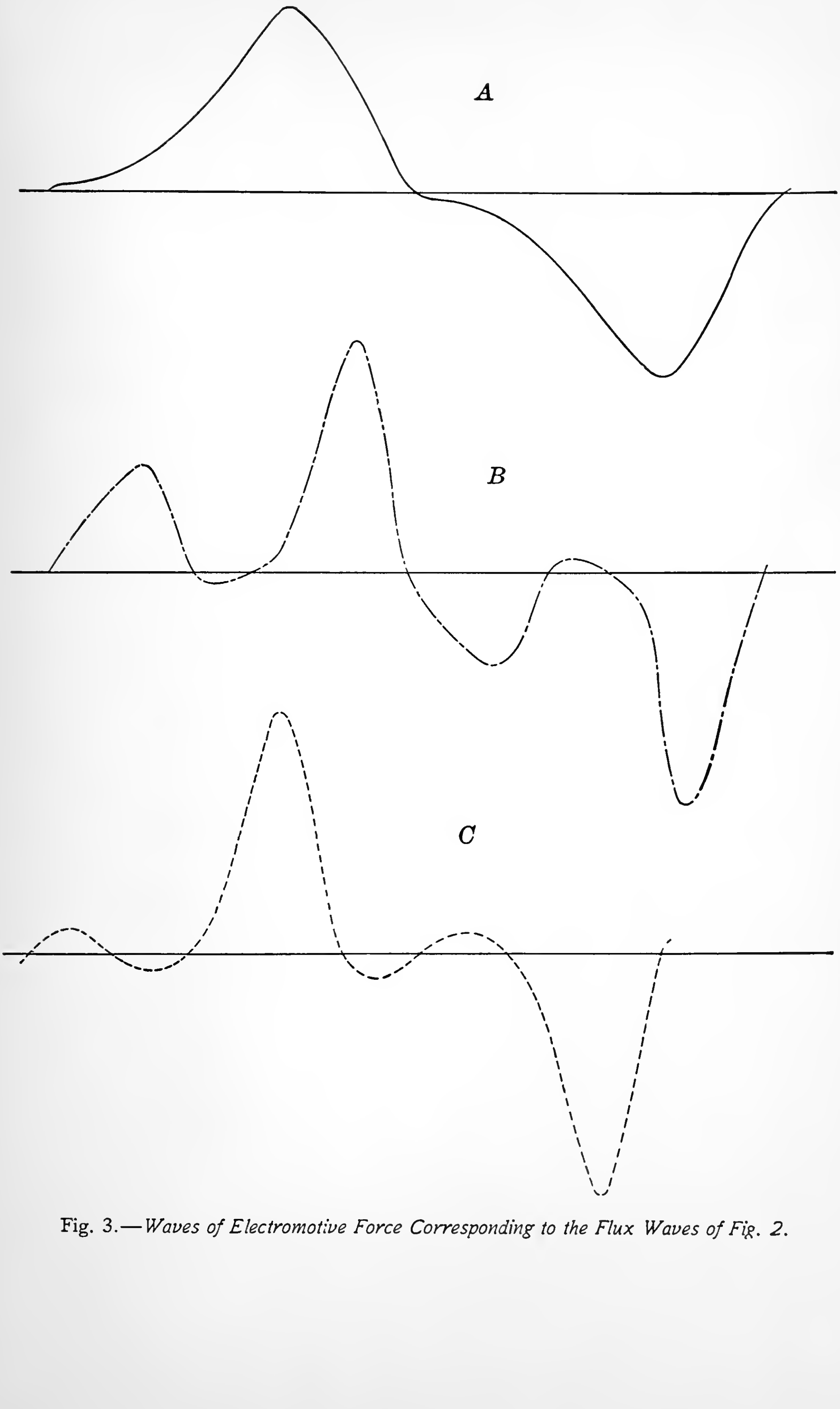

Fig. 3.—*Waves of Electromotive Force Corresponding to the Flux Waves of Fig. 2.*

If the terminals of the indicating instrument are reversed when the emf. is passing through zero there is no appreciable loss due to reversal; but if the reversal occurs at any other time, the reading will be not quite equal to the integrated value of the emf., since certain elements in the integration are lost during the time the circuit is broken during reversal. If the brush be made wide enough to bridge the gap on the commutator, so that one contact is made before the other is broken, the instrument is short-circuited during the overlap, and again the reading is too low. In the instrument described the breaks occupy 1.4 per cent of the time of a cycle, supposing the brushes to make a line contact. As the brushes are applied tangentially, with pressure, they spring into the air space and make the interval of break much less than this. Its exact value will depend upon the speed and has not been determined. For points near the maximum of the flux curve the elements lost are insignificant, and for points near the zero, where they become important, a larger percentage error may be tolerated.

Error may also arise from unequal spacing of the brushes, but it is not difficult to make this error negligible.

EXAMPLES OF USE.

Several examples of flux waves determined with this apparatus are shown in Fig. 2. The corresponding emf. waves induced in the surrounding coil, as determined with the oscillograph, are given in Fig. 3.

The wave form was altered by taking the magnetizing current from two generators in series, the frequencies being 60 and 180, respectively. The two generators have their shafts coupled together so that the speed and phase relation remains constant during any run. By varying the excitation of the two generators a wide variation in wave form is obtained; by reversing one of them, a dimpled emf. wave is changed to a peaked one, etc. Fig. 4 gives some additional examples of waves of magnetic flux.

In order to test the degree of accuracy obtainable, an iron ring, made up of a number of thin sheets, was used, and the wave form of the flux in the ring plotted out in the manner already described. The coil wound on the ring was then connected through a non-inductive resistance to the primary of a mutual inductance whose secondary in turn was connected to the rotating commutator, and a second set

of readings taken in order to secure the wave form of emf. The readings are given in Table I, the commutator advancing six degrees

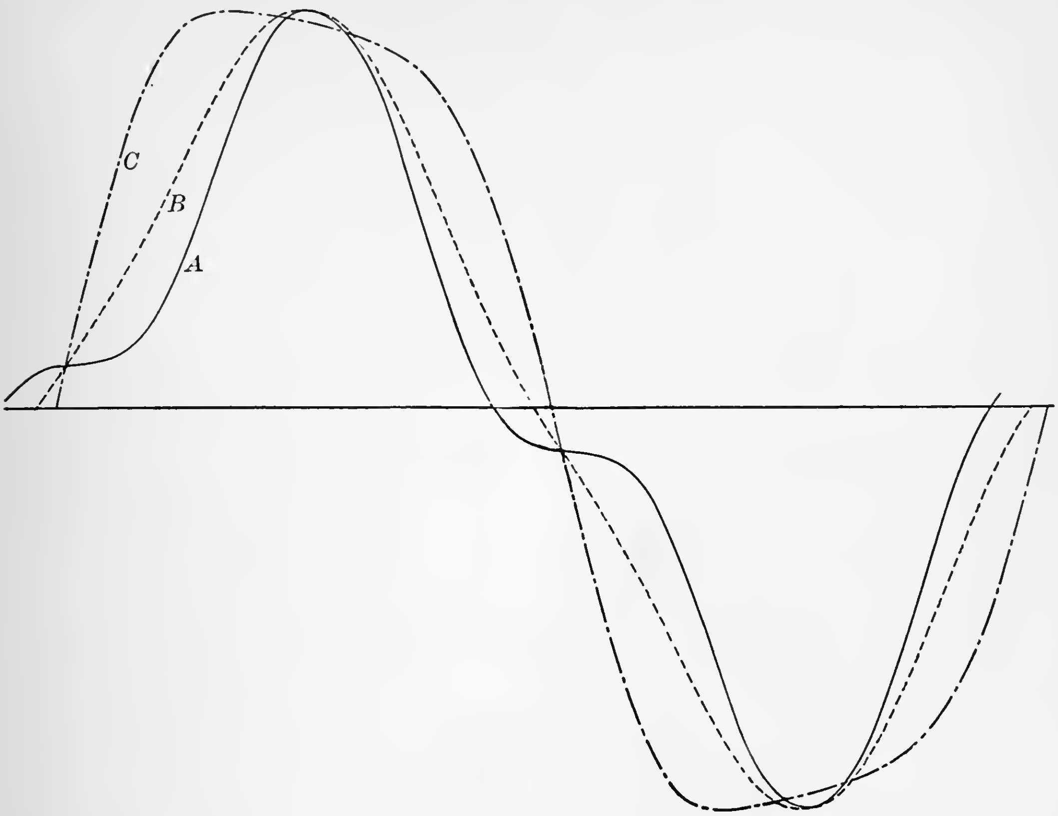

Fig. 4.—*Waves of Magnetic Flux. Nov. 11, 1907. Ring No. 31.* B_{max}=*10,000 gausses. Period = 0.0333 sec.*

each time; the curves plotted from these 15 equi-spaced ordinates are shown in Fig. 5.

Next each of the waves was analyzed into its component sine waves by a method given by Lyle,[1] and the proportions of the harmonics present, as well as the phase relations, were thus determined.

The constants of the circuits were as follows:

Mutual inductance, 0.0552 henry.

Resistance of primary of same, 212 ohms.

Resistance of secondary with voltmeter, 3560 ohms.

Resistance of voltmeter circuit when connected directly to coil, 33560 ohms.

Turns in coil, 315.

Maximum flux, 23500 maxwells.

Frequency, 30.

[1] T. R. Lyle, Phil. Mag., **11**, p. 25; 1906.

TABLE I.

Flux		Electromotive Force	
Degrees	Voltmeter Readings	Degrees	Voltmeter Readings
52	−351	−38	−712
46	− 17	−32	−571
40	+306	−26	−300
34	550	−20	− 33
28	689	−14	+104
22	738	− 8	+143
16	742	− 2	+118
10	726	+ 4	+ 39
4	721	10	− 19
− 2	753	16	− 16
− 8	812	22	+ 51
−14	873	28	+181
−20	903	34	+397
−26	843	40	+603
−32	650	46	+719

The equation found for the flux wave is

$$\Phi = 944 \sin a + 223 \sin 3 (a - 7^\circ 24') + 14 \sin 5 (a - 14^\circ 3') \\ + 3 \sin 7 (a + 19^\circ 40')$$

and for the emf.

$$e = 430 \sin (a - 90^\circ 20') + 295 \sin 3 (a - 38^\circ 41') \\ + 30 \sin 5 (a - 34^\circ 6') + 4.6 \sin 7 (a + 9^\circ 45')$$

where a is measured from the point where the fundamental of the flux wave crosses the axis, and the amplitudes are in divisions of the voltmeter and involve the constants of the circuits. To find whether the two waves are consistent, we make the amplitudes of the two fundamentals alike and then integrate the emf. wave.

$$\Phi = 1000 \sin a + 236 \sin 3 (a - 7^\circ 24') + 15 \sin 5 (a - 14^\circ 3') \\ + 3 \sin 7 (a + 19^\circ 40')$$

$$e = 1000 \sin (a - 90^\circ 20') + 686 \sin 3 (a - 38^\circ 41') + 70 \sin 5 (a - 34^\circ 6') + 11 \sin 7 (a + 9^\circ 45')$$

$$-\int e\, da = 1000 \cos (a - 90^\circ 20') + 229 \cos 3 (a - 38^\circ 41') + 14 \cos 5 (a - 34^\circ 6') + 2 \cos 7 (a + 9^\circ 45')$$
$$= 1000 \sin (a - 0^\circ 20') + 229 \sin 3 (a - 8^\circ 41') + 14 \sin 5 (a - 16^\circ 6') + 2 \sin 7 (a + 22^\circ 36')$$

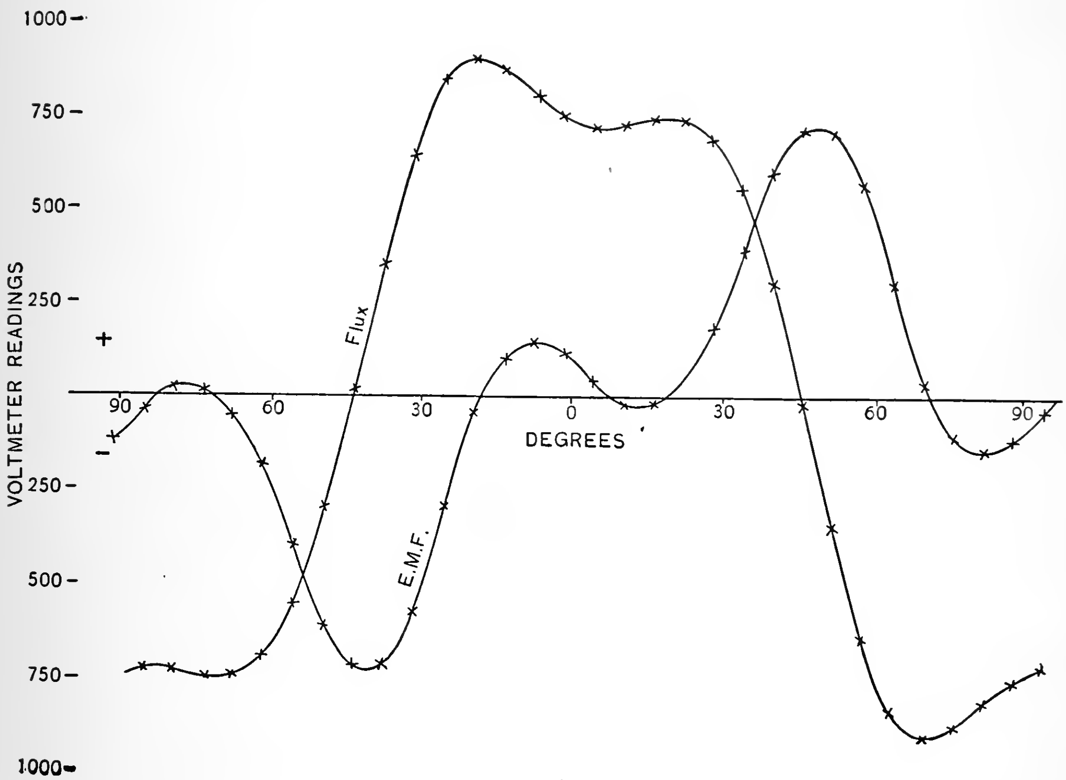

Fig. 5.

The accuracy of determination of the phase angle decreases as the amplitude decreases, so that it may be in error several degrees with the higher harmonics. This is especially true where only fifteen ordinates are used. The discrepancy in phase for the fundamental is only 20′, and if the fundamentals were brought into agreement, it would be less than one degree for the third harmonic. The principal discrepancy to be noted is in the amplitude of the third harmonic, amounting to seven units, or 0.7 per cent of the amplitude of the fundamental. An accuracy of better than one per cent is not to be expected from the present apparatus, as the brushes can be set only to the nearest 0.1°, and on a steep part of the curve this may cause an error in the reading of an ordinate, which would amount to one per cent of the maximum amplitude. In an

apparatus intended primarily for the accurate plotting of waves, means should be provided for a more accurate setting of the brushes.

Forty-five ordinates are read when greater accuracy is sought. The third, fifth, ninth, and fifteenth harmonics are then easily separated, and the seventh and any higher ones desired are afterward found in the usual way.

COMPARISON WITH PREVIOUS METHODS.

The first recorded waves of magnetic flux were computed by mechanical integration of the curve of secondary emf.

Sahulka[2] in 1898 described an apparatus similar to the present and based upon the same principle. He utilized, however, only one-half of the cycle, and his circuit was open seven-eighths of the time, and hence he did not obtain such steady readings on the voltmeter. This becomes of importance at low frequencies. Townsend[3] constructed a similar apparatus, utilizing half of each wave.

Lyle[4] has also utilized the same principle in a wave tracer, which may be used to determine the harmonics separately, as well as the total wave. He uses carbon brushes, and while no dimensions are given, the illustrations would indicate that the errors due to losing certain elements in the integration might be large. This source of error is not considered by him.

To Blondel[5] has been ascribed the idea of using an oscillograph whose galvanometer needle should have no control and no damping. The second differential of its angular position with respect to time would then be proportional to the instantaneous current. A coil surrounding the flux to be measured is connected to the primary of a transformer with air core, the secondary being connected to the galvanometer. In this way a current is secured which is proportional to the second differential of the flux with respect to time.

We have been unable to find any published description of such an apparatus or of its performance.

WASHINGTON, November 15, 1907.

---

[2] J. Sahulka. Zs. f. Elektrotechnik, **16**, p. 4; 1898.

[3] F. Townsend. American Inst. Electrical Engineers, Trans. **17**, p. 5; 1900.

[4] T. R. Lyle. Phil. Mag., **6**, p. 549; 1903.

[5] Electrician, **49**, p. 172; 1902.